# Matter-antimatter asymmetry without departure from thermal equilibrium

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We explore the possibility of baryogenesis without departure from thermal equilibrium. A possible scenario is found, though it contains strong constraints on the size of the $CPT$ violation ($CPTV$) effects and on the role of the $B$ (baryon number) nonconserving interactions which are needed for it.

In his seminal paper [1] Sakharov outlined three ingredients that are essential for an initially baryon-symmetric universe to dynamically evolve into one with a baryon asymmetry. These are: the presence of $B$ nonconserving interactions, violation of both $C$ and $CP$, and a departure from thermal equilibrium. It is clear that $B$ must be violated if the universe starts out as baryon symmetric and then generates a net baryon number $B$. Since the initial state with $B = 0$ is invariant under $C$ and $CP$, it will remain so, with the $B$-nonconserving reactions producing baryon and antibaryon excess at the same rate, unless both $C$ and $CP$ are violated. Finally, in thermal equilibrium particle phase space distributions are given by $f(p) = [\exp((E + \mu)/T) \pm 1]^{-1}$, and their densities by $n = \int d^3p\, f(p)/(2\pi)^3$. Here $E, \mu$ denote respectively the energy and the chemical potential of the particle. In chemical equilibrium the entropy is maximal when the chemical potentials associated with all non-conserved quantum numbers vanish, which implies that $\mu_b = \mu_{\bar{b}} = 0$. $CPT$ invariance ensures that $E^2 = p^2 + m^2$ and $m_b = m_{\bar{b}}$ for baryons and antibaryons, so that $n_b = n_{\bar{b}}$ unless there is a departure from thermal equilibrium.

The first two criteria of Sakharov are quite general. In the Standard Model (SM) baryon conservation is an accidental symmetry and one expects almost any extension of the SM to have $B$-nonconserving processes, which, however, have to be consistent with the strong limits on the mean lifetime $\tau$ of proton, $\tau(p \to e^+\pi^0) > 10^{33}$ y.

Both $C$ and $CP$ are observed to be violated microscopically. $C$ is maximally violated in the weak interactions, and both $C$ and $CP$ are violated in the interactions of $K^0$ and $\bar{K}^0$ mesons. Although a fundamental understanding of $CP$ violation is still lacking, without miraculous cancellations, the $CP$ violation in the neutral kaon sector will also lead to $CP$ violation in the $B$-nonconserving sector of any theory beyond the SM.

The third criterion, however, is more subtle. One has to note that the universe was already in thermal equilibrium very early (at least for $T \lesssim 10^{16}$ GeV, corresponding to time scales of about $10^{-38}$ sec, when the interactions mediated by photons occurred rapidly). If at that epoch the universe was still baryon symmetric, then one has to postulate a departure from thermal equilibrium and a subsequent return to it during the evolution, since matter (in the form of protons, electrons and hydrogen atoms) was in equilibrium with radiation for much of its early history until both decoupled at about $10^{13}$ sec.

This departure from thermal equilibrium is implemented through specific mechanisms. In GUT models the origin of the baryon asymmetry of the universe is explained through the existence of massive bosons whose interactions violate $B$ conservation. If their masses are sufficiently large ($> 10^{17}$ GeV), they will decay out of thermal equilibrium, producing a net baryon number. On the other hand, if inflation is produced after this baryogenesis, it would wash out the small asymmetry generated, and one should find alternative mechanisms to generate this.

Alternatives to GUT-baryogenesis have also been studied in recent years [2] through electroweak-baryogenesis, leptogenesis and Affleck-Dine baryogenesis, but it seems very difficult to generate asymmetry of the right order of magnitude in any model consistent with present phenomenology.

In view of these difficulties one wonders whether the third of Sakharov's criteria can be bypassed in some manner. Re-examining this condition it is evident that the only possibility would be to allow for a violation of $CPT$ [3, 4, 5]. In fact, it is possible to produce a large baryon asymmetry at the GUT scale through $CPT$-violating interactions [6].

In this letter we will follow this approach to relate baryogenesis to $CPTV$, and try to investigate whether

it is possible to generate the observed baryon asymmetry in thermal equilibrium at temperatures much below the GUT scale.

$CPT$ invariance is a fundamental symmetry of quantum field theory (QFT), which is the framework of present microscopic theories, in particular the SM. The difficulties in formulating a consistent QFT containing gravitation has led to questions about some of the underlying assumptions of QFT. For example, recent developments in quantum gravity [7] suggest that Lorentz invariance may not be an exact symmetry at high energies. $CPT$ conservation is also questioned within such contexts [8]. Recently $CPT$ violation has also been considered in connection with neutrino physics [9].

In summary, the possibility of $CPT$ violation is being considered quite extensively in recent years. One should, of course, note the most stringent limits on $CPT$ violation coming from kaon systems, $(M_{K^0} - M_{\bar{K}^0})/(M_{K^0} + M_{\bar{K}^0}) < 10^{-19}$, as well as from the leptonic sector, $(M_{e^+} - M_{e^-})/(M_{e^+} + M_{e^-}) < 4 \times 10^{-8}$.

In other words, any $CPT$ violating effect must necessarily be tiny. We are here interested in the corrections that such effects would produce in the calculation of the matter-antimatter densities ($n_b$-$n_{\bar{b}}$) in thermal equilibrium, possibly making $n_b \neq n_{\bar{b}}$ in the presence of $B$-violating interactions, namely, with zero chemical potential. Since the densities will depend on temperature, the correction will be temperature dependent. Therefore, we can parameterize this by a dimensionful parameter $\kappa$. If we assume that $\kappa$ has dimensions of energy in natural units, then

$$\frac{n_b - n_{\bar{b}}}{n_{\bar{b}}} = \frac{n_b}{n_{\bar{b}}} - 1 \sim \frac{\kappa}{T}, \tag{1}$$

where $\kappa \ll T$ and we assume that the particle mass ($m$) is much smaller than the temperature in order to neglect any $m/T$ dependence. We will call this an *infrared* (IR) effect of $CPT$ violation. The other possibility is that the parameter would have the dimension of $(\text{energy})^{-1}$. In fact this would seem more natural from the point of view of high-energy quantum gravity effects. The expected correction would then be of the form

$$\frac{n_b}{n_{\bar{b}}} - 1 \sim \ell T, \tag{2}$$

where we have taken a length scale $\ell$ as the parameter of this *ultraviolet* (UV) correction (in the context of quantum gravity, for example, $\ell$ can be the Planck length, $10^{-19}\,\text{GeV}^{-1}$). It is clear that this kind of correction would be less important at lower temperatures, so that it would not generate an asymmetry during the thermal evolution of the universe (in fact, such an effect could serve to symmetrize the abundance of particles and antiparticles at very early times if the initial conditions of the big-bang were not symmetric). Therefore, in order to generate a matter-antimatter asymmetry at lower temperatures, we have to assume an IR effect of $CPT$ violation. This may not be very unnatural and we will discuss briefly an explicit example later where such an IR effect does arise. We note that an IR scale correlated to an UV scale also arises naturally in non-commutative field theories [10], in large extra dimensions [11], and in considerations on entropy bounds [12].

At present the baryon density ($n_b$) is observed to be much larger than the antibaryon density ($n_b \gg n_{\bar{b}}$). Therefore one can use the approximation $n_B = n_b - n_{\bar{b}} \simeq n_b$. The baryon to photon ratio $\eta \equiv n_b/n_\gamma$ is estimated from direct measurements to be around $10^{-9}$, which agrees with the value needed for the primordial nucleosynthesis. The number of photons in the universe has not remained constant, but has increased at various epochs when particle species have annihilated (e.g. $e^\pm$ pairs at $T \simeq 0.5\,\text{MeV}$). However, as in standard cosmology [13], we assume that there has not been significant entropy production during the expansion (adiabatic expansion), so that the entropy per comoving volume ($\propto sR^3$) has remained constant. This is also the case for the baryon number per comoving volume ($\propto n_B R^3 \propto n_B/s$) in the absence of $B$-nonconserving interactions (or if they occur very slowly). Since the entropy density is related to the density of photons through the effective number of degrees of freedom $g_*$ at any temperature as $s \simeq g_* n_\gamma$, we have at present

$$\frac{n_B}{s} \simeq \frac{1}{7}\eta \simeq 10^{-10}. \tag{3}$$

As long as the expansion is isentropic and the baryon number is at least effectively conserved this ratio remains constant.

Eq. (1) applied to the quark-antiquark asymmetry implies that a baryon asymmetry can be generated during the evolution of the universe even in thermal equilibrium (in the presence of $B$-nonconserving interactions). Prior to $10^{-6}$ sec after the big-bang, quarks and antiquarks were in thermal equilibrium with photons, and $n_q \simeq n_{\bar{q}} \simeq n_\gamma$, so that

$$\frac{n_q - n_{\bar{q}}}{n_{\bar{q}}} \simeq \frac{n_B}{3n_\gamma} \simeq g_* \frac{n_B}{3s}. \tag{4}$$

Since $g_* \simeq 10^2$ for $T \gtrsim 1\,\text{GeV}$,

$$\frac{n_B}{3s} \simeq 10^{-11} \left(\frac{\kappa}{\text{eV}}\right) \left(\frac{\text{GeV}}{T}\right). \tag{5}$$

If $B$-nonconserving interactions decouple below a temperature $T_D$, the value of $\kappa$ necessary to reproduce the observed baryon asymmetry is

$$\frac{\kappa}{\text{eV}} \simeq \frac{10}{3} \frac{T_D}{\text{GeV}}. \tag{6}$$

If one considers $B$-nonconserving interactions down to $T_D \lesssim 1$ GeV then one has to go beyond the simple approximation in Eq. (1) incorporating the dependence on masses which we do not consider in this paper. The phenomenological consistency of the presence of $B$-violating interactions down to temperatures much below the GUT scale, which is an assumption implicit in this work, is an interesting question which has been discussed in the context of several alternatives to GUT-baryogenesis. The discussion of this problem in an extension of QFT with $CPTV$ interactions goes beyond the scope of the present work.

We will now give an example of an extension of QFT where one can explicitly demonstrate the infrared effects arising from $CPT$ violation [14]. The extension is based on a generalization of the canonical commutation relations. The simplest example one can consider is the theory of a free complex scalar noncommutative field defined by the Hamiltonian

$$H = \frac{1}{2}\sum_{i=1}^{2}\int \mathrm{d}^3x \left[\pi_i^2 + (\nabla\phi_i)^2 + m^2\phi_i^2\right], \tag{7}$$

and commutators

$$\begin{aligned} [\pi_i(\mathbf{x}), \pi_j(\mathbf{x}')] &= \epsilon_{ij}\mathcal{B}\delta(\mathbf{x}-\mathbf{x}'), &(8)\\ [\phi_i(\mathbf{x}), \phi_j(\mathbf{x}')] &= \epsilon_{ij}\theta\delta(\mathbf{x}-\mathbf{x}'), &(9)\\ [\phi_i(\mathbf{x}), \pi_j(\mathbf{x}')] &= i\,\delta_{ij}\,\delta(\mathbf{x}-\mathbf{x}'), &(10)\end{aligned}$$

where $\mathcal{B}$ and $\theta$ characterizing the deformation of the canonical commutation relations carry dimensions of energy and length respectively.

In [14] it has been shown that (7-10) lead to an anisotropic quantum field theory in the sense that the second quantized Hamiltonian can be written in the diagonal form as

$$\begin{aligned} H &= \int \frac{\mathrm{d}^3p}{(2\pi)^3}\left[E(\mathbf{p})\left(a^\dagger_{\mathbf{p}}a_{\mathbf{p}} + \frac{1}{2}\right)\right.\\ &+ \left.\bar{E}(\mathbf{p})\left(b^\dagger_{\mathbf{p}}b_{\mathbf{p}} + \frac{1}{2}\right)\right], \end{aligned}\tag{11}$$

where $E(\mathbf{p})$ and $\bar{E}(\mathbf{p})$ are given by

$$\begin{aligned} E(\mathbf{p}) &= \omega(\mathbf{p})\left[\sqrt{1+\frac{1}{4}\left(\frac{\mathcal{B}}{\omega(\mathbf{p})} - \theta\omega(\mathbf{p})\right)^2}\right.\\ &- \left.\frac{1}{2}\left(\frac{\mathcal{B}}{\omega(\mathbf{p})} + \theta\omega(\mathbf{p})\right)\right], &(12)\\ \bar{E}(\mathbf{p}) &= \omega(\mathbf{p})\left[\sqrt{1+\frac{1}{4}\left(\frac{\mathcal{B}}{\omega(\mathbf{p})} - \theta\omega(\mathbf{p})\right)^2}\right.\\ &+ \left.\frac{1}{2}\left(\frac{\mathcal{B}}{\omega(\mathbf{p})} + \theta\omega(\mathbf{p})\right)\right], &(13)\end{aligned}$$

and

$$\omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}. \tag{14}$$

Thus we see that the free theory of the noncommutative scalar field is a quantum field theory where the symmetry between particles and antiparticles is lost. This is, of course, a consequence of the violation of Lorentz invariance which is manifest in the Lagrangian description.

When the momentum of the particle $\mathbf{p}$ is such that $\mathcal{B} \ll \omega(\mathbf{p}) \ll \theta^{-1}$ then one has $E(\mathbf{p}) \approx \bar{E}(\mathbf{p}) \approx \omega(\mathbf{p})$ and one recovers the standard relativistic theory with a particle-antiparticle symmetry. This symmetry, however, is lost both in the high energy limit $\omega(\mathbf{p}) \sim \theta^{-1}$ and in the low energy limit $\omega(\mathbf{p}) \sim \mathcal{B}$.

Let us next show that this simple theory gives an explicit realization of an asymmetry between particles and antiparticles due to $CPT$ violation in the infrared. Let us consider a system of the two types of particles in thermodynamical equilibrium at temperature $T$. The number of particles of each type in a volume $V$ is given by (we have set $\mu = \bar{\mu} = 0$ in anticipation that the fully interacting theory would have "$B$ violation")

$$n = 4\pi V\int_0^\infty \frac{p^2\mathrm{d}p}{\left[e^{\frac{E}{T}} - 1\right]}, \quad \bar{n} = 4\pi V\int_0^\infty \frac{p^2\mathrm{d}p}{\left[e^{\frac{\bar{E}}{T}} - 1\right]}. \tag{15}$$

If we consider a temperature $T$ such that $\theta T \ll \mathcal{B}/T \ll m/T \ll 1$, then one has a tiny asymmetry arising from (15) due to the infrared scale $\mathcal{B}$,

$$\frac{n}{\bar{n}} - 1 \approx \alpha\,\frac{\mathcal{B}}{T}, \tag{16}$$

where we have neglected higher order terms in an expansion in powers of $\mathcal{B}/T$ as well as corrections due to the ultraviolet scale $\theta^{-1}$ and the mass. The coefficient of the linear term, $\alpha$, has the value

$$\alpha = \frac{\displaystyle\int_0^\infty \frac{e^{\frac{p}{T}}p^2\mathrm{d}p}{\left[e^{\frac{p}{T}} - 1\right]^2}}{\displaystyle\int_0^\infty \frac{p^2\mathrm{d}p}{\left[e^{\frac{p}{T}} - 1\right]}} = \frac{\zeta(2)}{\zeta(3)} \approx 1. \tag{17}$$

The result in (16) can be compared with the expression (1) for the baryon asymmetry induced by $CPTV$ in the infrared and leads to the identification

$$\kappa = \alpha\,\mathcal{B} \simeq \mathcal{B}. \tag{18}$$

This shows that a very simple extension of QFT has the necessary ingredients to generate a matter-antimatter asymmetry induced by $CPTV$. In order to have a realistic model one should go beyond the free theory and incorporate interactions violating baryon number.

We can also use this simple model to comment on the relation between this asymmetry and the mass difference between the particle and the antiparticle. It is not clear how to define the mass of a particle when Lorentz invariance is violated. One can consider the effect of the

infrared scale $\mathcal{B}$ on the kinematic analysis of any process. If one considers processes where the number of particles minus antiparticles remains constant (i.e., if one neglects interactions violating the $U(1)$ symmetry of the free theory of the complex field) then one can easily see from the expressions in (12)-(13) that the only kinematic effect of the noncommutative parameter $\mathcal{B}$ is to replace $m^2$ by $m^2 + \mathcal{B}^2/4$. In this case, the only difference from the conventional relativistic kinematic analysis is a lower bound ($\mathcal{B}^2/4$) on the mass squared, but there is no reflection of the $CPTV$ of the free theory at the level of a mass difference between particles and antiparticles. In the presence of interactions violating the $U(1)$ symmetry (which we have assumed implicitly), however, the theory will generate small mass differences of the order of $g^2\mathcal{B}$ for a weak coupling $g$ of such interactions. This illustrates how a particle-antiparticle asymmetry can be generated through $CPTV$ independent of the mass difference between particles and antiparticles which is necessary in any attempt to ascribe matter-antimatter asymmetry of our Universe to $CPTV$ because of the very stringent experimental limits on $CPT$.

In conclusion, the considerations of $CPTV$ effects, which have started to be taken seriously in recent years, lead naturally to a critical reevaluation of the third criterion of Sakharov for baryogenesis. We find that the generation of a net baryon number may be possible without departure from thermal equilibrium, with considerable restrictions on the size of $CPTV$ effects and the temperature at which $B$-nonconserving interactions stop being relevant. In this scenario, one can reformulate the criteria for the observable matter-antimatter asymmetry as (a) the presence of $B$-violating processes down to an energy scale much lower than what is commonly assumed in GUT models; (b) $C$ and $CP$ violation; (c) $CPTV$ parametrized by an infrared scale ($\kappa$) which is of the order of a few eV if $B$-nonconserving interactions extend down to temperatures below the nucleon mass, of the order of a KeV if such processes decouple at $T_D \sim 100\,\mathrm{GeV}$ and proportional to $T_D$ for higher values (see Eq. (6)).

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